If you haven't read parts I and II of this topic, I strongly suggest you read them first:Bagging the Graphs – Part I, Bagging the Graphs – Part II

While introducing this concept in Part I of Graphs, I had mentioned: "the one thing that I would suggest to increase speed in Co-ordinate Geometry and Algebra is Graphs"

Did you wonder why I included "Algebra" here? If yes, then this post will answer your question. In part 2, I gave an example of a Geometry question that can be easily solved using Graphs. In this post, I will take up an Algebra question for which you can do the same.

In part I of Graphs, I had also mentioned "Learn how to draw a line from its equation in under ten seconds and you shall solve the related question in under a minute." After this post, you won't have to take my word for it!

Before I begin, let me also add here that this Data Sufficiency question is not a question I created. So don't think I made it to conveniently suit my needs. It is a question someone asked me on a GMAT forum and was created by some third party. I chose it to demonstrate the beauty of graphs to you.

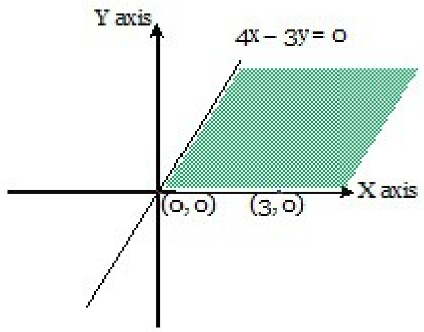
Question: If x and y are positive, is $4x > 3y$?

Statements:

1. $x > y – x$
2. $x/y < 1$

Let us look at the question stem first: x and y are positive, 'is $4x > 3y$?' or rephrase it as 'is $4x – 3y > 0$?'

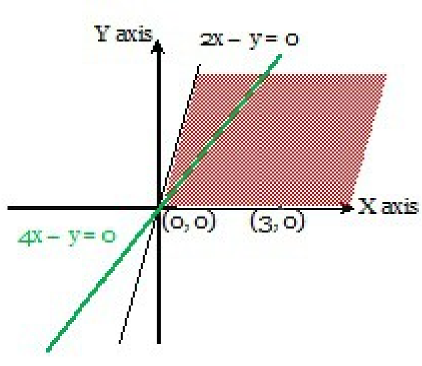
Let us draw $4x – 3y = 0$. Then we can figure out which region represents $4x – 3y > 0$. When $x = 0$, $y = 0$ so the line passes through (0, 0). The slope of the line is 4/3. This is what it looks like:



The line has divided the graph into two regions: $4x – 3y < 0$ and $4x – 3y > 0$. Let us check in which region, the point (3, 0) lies. (This is an arbitrary choice. You can check for any point.) When you put $x = 3$ and $y = 0$ in $4x – 3y > 0$, you get $12 > 0$ which is true. Hence the point (3, 0) lies in $4x – 3y > 0$ region. Since x and y are positive, we are only concerned with quadrant I. The shaded Green region is where $4x – 3y > 0$. So the question boils down to: "Does the point (x, y) lie in the shaded Green region for all values of x and y?"

Statement 1: $x > y – x$ or $2x – y > 0$

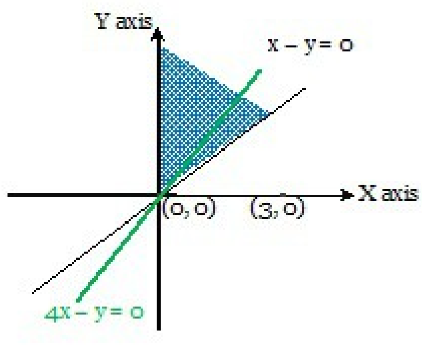
Draw $2x – y = 0$. When $x= 0$, $y = 0$ so this line passes through the center. The slope of the line is 2. The slope of this line, 2, is greater than 4/3, the slope of the line drawn above. Hence, this line will be steeper than the line drawn above. Check for point (3, 0) again to find whether it lies in region $2x – y > 0$. Putting $x = 3$ and $y = 0$, we get $6 > 0$ which is true so the region $2x – y > 0$ includes the point (3, 0) and is as shown below:



The Red shaded region here includes all the points of the Green shaded region above plus some more. Hence all points of Red region may not lie in the Green region. Therefore, if values of x and y satisfy $2x – y > 0$, they may or may not satisfy $4x – 3y > 0$. Hence statement 1 alone is not sufficient.

Statement 2: $x/y < 1$

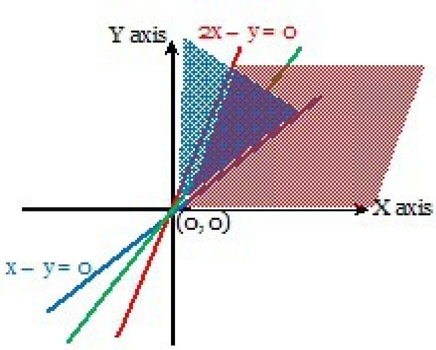
Since x and y are positive, we can multiply both sides of the inequality by y to get $x < y$ or $x – y < 0$. Draw $x – y = 0$. When $x= 0$, $y = 0$ so this line passes through the center. The slope of the line is 1. The slope of this line, 1, is less than 4/3, the slope of the line in the question. Hence, this line will be less steep than the line in the question stem. Check for point (3, 0) again to find whether it lies in region $x – y < 0$. Putting $x = 3$ and $y = 0$, we get $3 < 0$ which is not true so the region $x – y < 0$ does not include the point (3, 0) and is as shown below:



Note: Our concern is limited to first quadrant since x and y are both positive.

The Blue shaded region here includes some points of the Green shaded region above plus some more. Hence all points of Blue region may not lie in the Green region. Therefore, if values of x and y satisfy $x – y < 0$, they may or may not satisfy $4x – 3y > 0$. Hence statement 2 alone is not sufficient.

Taking both statements together, x and y will have values that overlap in the Red and the Blue region as shown in the graph below. Some of these values will lie in the Green shaded region above, some will not.



So even if we take both statements together, they are not sufficient. Answer (E).

I hope you have come to appreciate the wide range of applicability of graphs. Next time, I will introduce a graphical way of working with Modulus and Inequalities.